$$Y_{l,m_l}(\theta,\phi) = \Theta(\theta)\,\Phi(\phi)$$

$$E = \frac{\hbar^2}{2I}\,l(l+1)$$

If the energy is fully kinetic, $V=0$

$$E = \frac{1}{2} I\omega^2$$

$$L = I\omega$$

$$E = \frac{\hat{L}^2}{2I}$$

Particle in a 1-D box, $E = \frac{\hat{p}_x^2}{2m}$

$$-\frac{\hbar^2}{2I}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta} + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right]Y_{l,m_l}$$

$$= E\,Y_{l,m_l}(\theta,\phi) \quad \ldots \quad (1)$$

$$\frac{\hat{L}^2}{2I}Y_{l,m_l} = E\,Y_{l,m_l}(\theta,\phi) \quad \ldots \quad (2)$$

$$\hat{L}^2 = -\hbar^2\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta} + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right]$$

$$= -\hbar^2\Lambda^2$$

$$\frac{\hat{L}^2}{2I}Y_{l,m_l} = \frac{\hbar^2}{2I}\,l(l+1)\,Y_{l,m_l}(\theta,\phi)$$

$$\hat{L}^2 Y_{l,m_l}(\theta,\phi) = \underbrace{\hbar^2 l(l+1)}_{\text{eigen value}} Y_{l,m_l}(\theta,\phi) \quad \cdots (3)$$

$$\hat{L}^2 = \hat{L}_x^2 + \hat{L}_y^2 + \hat{L}_z^2$$

$$\hat{L}_z Y_{l,m_l}(\theta,\phi) = m_l \hbar Y_{l,m_l}(\theta,\phi) \quad \cdots (4)$$

$$\hat{L}_z = -i\hbar \frac{\partial}{\partial \phi}$$

$$(\underbrace{\hat{L}^2 - \hat{L}_z^2}_{\hat{L}_x^2 + \hat{L}_y^2}) Y_{l,m_l}(\theta,\phi) = \left[\hbar^2 l(l+1) - m_l^2 \hbar^2\right] Y_{l,m_l}(\theta,\phi) \quad \cdots (5)$$

$$\begin{tikzcd} & L_x \arrow[dr] & \\ L_z \arrow[ur] & & L_y \arrow[ll] \end{tikzcd}$$

$$[\hat{L}_x, \hat{L}_y] = i\hbar \hat{L}_z$$

$$[\hat{L}_y, \hat{L}_z] = i\hbar \hat{L}_x$$

$$[\hat{L}_z, \hat{L}_x] = i\hbar \hat{L}_y$$

$$[\hat{L}^2, \hat{L}_x] = [\hat{L}^2, \hat{L}_y] = [\hat{L}^2, \hat{L}_z] = 0$$

$$\hat{L}^2 Y_{l,m_l} = \hbar^2 l(l+1) Y_{l,m_l}$$

$$|L| = \hbar \sqrt{l(l+1)}$$

$$\cos\theta = \frac{m_l \hbar}{\hbar\sqrt{l(l+1)}}$$

$$\cos\theta = \frac{m_l}{\sqrt{l(l+1)}}$$ - - - - space quantization

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta = N\cos\theta$$

$$\Lambda^2 N\cos\theta$$

$$\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta} + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right] N\cos\theta$$

$$\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta} N\cos\theta$$

$$= -\frac{1}{\sin\theta}\frac{\partial}{\partial\theta} N\sin^2\theta$$

$$= -\frac{1}{\sin\theta} 2N\sin\theta\cos\theta$$

$$= \underline{-2N\cos\theta}$$

$$E_J = \frac{\hbar^2}{2I} J(J+1)$$

energy levels of a rotating molecule

$$\Delta E = E_{J+1} - E_J$$

$$= \frac{\hbar^2}{2I} \left[(J+1)(J+2) - J(J+1) \right]$$

$$= \frac{\hbar^2}{2I} 2(J+1)$$

$$= \frac{\hbar^2}{I} (J+1)$$