

$$Y_{l,m_l}(\theta, \phi) = \Theta(\theta) \Phi(\phi)$$

$$E = \frac{\hbar^2}{2I} l(l+1)$$

If the energy is fully kinetic,  $V=0$

$$E = \frac{1}{2} I \omega^2$$

$$L = I\omega$$

$$E = \frac{\hat{L}^2}{2I}$$

Particle in a 1-D box,  $E = \frac{\hat{p}_x^2}{2m}$

$$-\frac{\hbar^2}{2I} \left[ \frac{1}{\sin\theta} \frac{\partial}{\partial\theta} \sin\theta \frac{\partial}{\partial\theta} + \frac{1}{\sin^2\theta} \frac{\partial^2}{\partial\phi^2} \right] Y_{l,m_l}$$

$$= E Y_{l,m_l}(\theta, \phi) \quad \dots \quad (1)$$

$$\frac{\hat{L}^2}{2I} Y_{l,m_l} = E Y_{l,m_l}(\theta, \phi) \quad \dots \quad (2)$$

$$\hat{L}^2 = -\hbar^2 \left[ \frac{1}{\sin\theta} \frac{\partial}{\partial\theta} \sin\theta \frac{\partial}{\partial\theta} + \frac{1}{\sin^2\theta} \frac{\partial^2}{\partial\phi^2} \right]$$

$$= -\hbar^2 \Lambda^2$$

$$\frac{\hat{L}^2}{2I} Y_{l,m_l} = \frac{\hbar^2}{2I} l(l+1) Y_{l,m_l}(\theta, \phi)$$

$$\hat{L}^2 Y_{l,m_l}(\theta, \phi) = \hbar^2 l(l+1) Y_{l,m_l}(\theta, \phi) \quad \text{--- (3)}$$

$\downarrow$   
eigen value

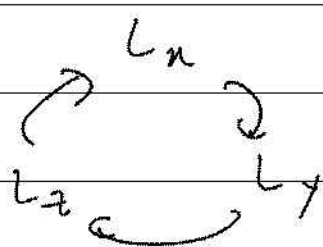
$$\hat{L}^2 = \hat{L}_x^2 + \hat{L}_y^2 + \hat{L}_z^2$$

$$\hat{L}_z Y_{l,m_l}(\theta, \phi) = m_l \hbar Y_{l,m_l}(\theta, \phi) \quad \text{--- (4)}$$

$$\hat{L}_z = -i\hbar \frac{\partial}{\partial \phi}$$

$$(\hat{L}^2 - \hat{L}_z^2) Y_{l,m_l}(\theta, \phi) = [\hbar^2 l(l+1) - m_l^2 \hbar^2] Y_{l,m_l}(\theta, \phi) \quad \text{--- (5)}$$

$\hat{L}_x^2 + \hat{L}_y^2$



$$[\hat{L}_x, \hat{L}_y] = i\hbar \hat{L}_z$$

$$[\hat{L}_y, \hat{L}_z] = i\hbar \hat{L}_x$$

$$[\hat{L}_z, \hat{L}_x] = i\hbar \hat{L}_y$$

$$[\hat{L}^2, \hat{L}_x] = [\hat{L}^2, \hat{L}_y] = [\hat{L}^2, \hat{L}_z] = 0$$

$$\hat{L}^2 Y_{l,m_l} = \hbar^2 l(l+1) Y_{l,m_l}$$

$$|L| = \hbar \sqrt{l(l+1)}$$

$$\cos \theta = \frac{m_l \hbar}{k \sqrt{l(l+1)}}$$

$$\cos \theta = \frac{m_l}{\sqrt{l(l+1)}} \quad \text{--- space quantization}$$

$$Y_1^0 = \sqrt{\frac{3}{4\pi}} \cos \theta = N \cos \theta$$

$$N^2 N \cos \theta$$

$$\left[ \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \sin \theta \frac{\partial}{\partial \theta} + \frac{1}{\sin^2 \theta} \frac{\partial^2}{\partial \phi^2} \right] N \cos \theta$$

$$\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \sin \theta \frac{\partial}{\partial \theta} N \cos \theta$$

$$= - \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} N \sin^2 \theta$$

$$= - \frac{1}{\sin \theta} 2 N \sin \theta \cos \theta$$

$$= \underline{\underline{-2 N \cos \theta}}$$

$$E_J = \frac{\hbar^2}{2I} J(J+1)$$

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↑  
energy levels of a rotating  
molecule

$$\begin{aligned}\Delta E &= E_{J+1} - E_J \\ &= \frac{\hbar^2}{2I} [(J+1)(J+2) - J(J+1)] \\ &= \frac{\hbar^2}{2I} 2(J+1) \\ &= \frac{\hbar^2}{I} (J+1)\end{aligned}$$

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